Problem 6.15: Digital and Analog Speech Communication
Suppose we transmit speech signals over comparable digital and analog channels. We want to compare the resulting quality of the received signals. Assume the transmitters use the same power, and the channels introduce the same attenuation and additive white noise. Assume the speech signal has a 4 kHz bandwidth and, in the digital case, is sampled at an 8 kHz rate with eight-bit A/D conversion. Assume simple binary source coding and a modulated BPSK transmission scheme.

(a) What is the transmission bandwidth of the analog (AM) and digital schemes?
(b) Assume the speech signal’s amplitude has a magnitude less than one. What is maximum amplitude quantization error introduced by the A/D converter?
(c) In the digital case, each bit in quantized speech sample is received in error with probability \( p_e \) that depends on signal-to-noise ratio \( \frac{E_b}{N_0} \). However, errors in each bit have a different impact on the error in the reconstructed speech sample. Find the mean-squared error between the transmitted and received amplitude.
(d) In the digital case, the recovered speech signal can be considered to have two noise sources added to each sample’s true value: One is the A/D amplitude quantization noise and the second is due to channel errors. Because these are separate, the total noise power equals the sum of these two. What is the signal-to-noise ratio of the received speech signal as a function of \( p_e \)?
(e) Compute and plot the received signal’s signal-to-noise ratio for the two transmission schemes for a few values of channel signal-to-noise ratios.
(f) Compare and evaluate these systems.

Problem 6.16: Source Compression
Consider the following 5-letter source.

<table>
<thead>
<tr>
<th>Letter</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.5</td>
</tr>
<tr>
<td>b</td>
<td>0.25</td>
</tr>
<tr>
<td>c</td>
<td>0.125</td>
</tr>
<tr>
<td>d</td>
<td>0.0625</td>
</tr>
<tr>
<td>e</td>
<td>0.0625</td>
</tr>
</tbody>
</table>

Table 6.5

(a) Find this source’s entropy.
(b) Show that the simple binary coding is inefficient.
(c) Find an unequal-length codebook for this sequence that satisfies the Source Coding Theorem. Does your code achieve the entropy limit?
(d) How much more efficient is this code than the simple binary code?

Problem 6.18: Speech Compression
When we sample a signal, such as speech, we quantize the signal’s amplitude to a set of integers. For a \( b \)-bit converter, signal amplitudes are represented by \( 2^b \) integers. Although these integers could be represented by a binary code for digital transmission, we should consider whether a Huffman coding would be more efficient.

(a) Load into Matlab the segment of speech contained in \( y.mat \). Its sampled values lie in the interval \((-1, 1)\). To simulate a 3-bit converter, we use Matlab’s round function to create quantized amplitudes corresponding to the integers \([0 1 2 3 4 5 6 7]\).

\[ y_{\text{quant}} = \text{round}(3.5*y + 3.5); \]
Find the relative frequency of occurrence of quantized amplitude values. The following Matlab program computes the number of times each quantized value occurs.

```matlab
for n=0:7; count(n+1) = sum(y_quant == n); end;
```

Find the entropy of this source.

(b) Find the Huffman code for this source. How would you characterize this source code in words?

(c) How many fewer bits would be used in transmitting this speech segment with your Huffman code in comparison to simple binary coding?

Problem 6.22: Error Correcting Codes
A code maps pairs of information bits into codewords of length-5 as follows.

<table>
<thead>
<tr>
<th>Data</th>
<th>Codeword</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>00000</td>
</tr>
<tr>
<td>01</td>
<td>01101</td>
</tr>
<tr>
<td>10</td>
<td>10111</td>
</tr>
<tr>
<td>11</td>
<td>11010</td>
</tr>
</tbody>
</table>

Table 6.9

(a) What is this code’s efficiency?
(b) Find the generator matrix $G$ and parity-check matrix $H$ for this code.
(c) Give the decoding table for this code. How many patterns of one, two, and three errors are correctly decoded?
(d) What is the block error probability (the probability of any number of errors occurring in the decoded codeword)?

Problem 6.28: Communication System Design
RU Communication Systems has been asked to design a communication system that meets the following requirements.

- The baseband message signal has a bandwidth of 10 kHz.
- The RUCS engineers find that the entropy $H$ of the sampled message signal depends on how many bits $b$ are used in the A/D converter (see table below).
- The signal is to be sent through a noisy channel having a bandwidth of 25 kHz centered at 2 MHz and a signal-to-noise ratio within that band of 10 dB.
- Once received, the message signal must have a signal-to-noise ratio of at least 20 dB.

<table>
<thead>
<tr>
<th>$b$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.19</td>
</tr>
<tr>
<td>4</td>
<td>3.25</td>
</tr>
<tr>
<td>5</td>
<td>4.28</td>
</tr>
<tr>
<td>6</td>
<td>5.35</td>
</tr>
</tbody>
</table>

Table 6.13

Can these specifications be met? Justify your answer.

Problem 6.32: Repeaters
Because signals attenuate with distance from the transmitter, *repeaters* are frequently employed for both analog and digital communication. For example, let’s assume that the transmitter and receiver are $D$ m apart, and a repeater is positioned halfway between them (Figure 6.42). What the repeater does is amplify its received signal to exactly cancel the attenuation encountered along the first leg and to re-transmit the signal to the ultimate receiver. However, the signal the repeater receives contains white noise as well as the transmitted signal. The receiver experiences the same amount of white noise as the repeater.
(a) What is the block diagram for this system?
(b) For an amplitude-modulation communication system, what is the signal-to-noise ratio of the demodulated signal at the receiver? Is this better or worse than the signal-to-noise ratio when no repeater is present?
(c) For digital communication, we must consider the system’s capacity. Is the capacity larger with the repeater system than without it? If so, when; if not, why not?