\[ H(A) = \frac{1}{4} \cdot 2 + 4 \cdot \frac{1}{8} \cdot 3 + 4 \cdot \frac{1}{16} \cdot 4 = \frac{1}{2} + \frac{3}{2} + 1 = 3 \text{ bits} \]

Average code length has the same formula. Maximally efficient!

b) Note that the sum of the last two columns has only 2 ones. \[ G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \]

Has 5 distinct columns \( \Rightarrow \) can correct 5 single bit error patterns

The remaining one will not be detected or corrected.

c) \[ C = 1 + \frac{1}{8} \log_2 \frac{1}{8} + \frac{3}{8} \log_2 \frac{7}{8} \]
\[ = 1 - \frac{3}{8} - \frac{7}{8} (3 - \log_2 7) = 0.456 \text{ bits} \]

Code needs to have a rate \( \frac{K}{N} \leq 0.456 \). Even if code above worked, it is a rate \( \frac{1}{2} \) code \( \Rightarrow \) not good enough.