a) Half-wave rectify and lowpass filter. Positive & negative parts.

\[ X(t) \xrightarrow{\text{H} \text{U} \text{R}} L \text{P} \text{F} \rightarrow M_1(t) \quad \text{Incoherent Demodulator} \]

\[ \text{Incoherent Demodulator} \]

b) Positive signal can be written as

\[ A \left[ \pm \text{a.m.} \left( t \right) \right] \text{H} \text{U} \text{R} \left( \sin(2\pi f_c t) \right) \]

Sine-cosine Fourier series for \( \text{H} \text{U} \text{R} \left( \sin (2\pi f_c t) \right) \)

\[ a_1 = \frac{1}{2}, \quad a_2 = 0 = a_3 = \ldots \]

\[ g_k = \left\{ \begin{array}{ll}
\frac{k}{2} & k = 1, 4, \ldots \text{L.odd} \\
0 & \text{otherwise}
\end{array} \right. \quad \Rightarrow C_k = \left\{ \begin{array}{ll}
0 & k = 0 \\
-\frac{1}{4} & k = 1 \\
\frac{1}{2} & k = 2, 4, 6, \ldots \\
0 & k = 3, 5, 7, \ldots
\end{array} \right. \]

Spectrum equals

\[ \sum_{k=-\infty}^{\infty} C_k M(f - k f_c) \quad C_k: \text{Coefficient of} \ \text{H} \text{U} \text{R} \left( \sin(2\pi f_c t) \right) \]

\[ f \]

\[ f_c \]

\[ k f_c \]
c) Receiver can be found. What about transmission bandwidth?

Spectrum of negative portion needs to be explored.

\[ A[1 + a m_2(t)] H_{\text{IR}}(-\sin(2\pi f_c t)) \]

Fourier series for \( H_{\text{IR}}(-\sin(2\pi f_c t)) = \sum C_k e^{-j\pi k} \)

\[ C_k = C_k e^{-j\pi k} = C_k (-1)^k \]

Consequently, spectrum of \( x(t) \) \([m_1(t) + m_2(t) \text{ sinusoids}]\)

\[ |x(f)| \]

\[ f_c \quad 2f_c \quad 3f_c \quad 4f_c \]

Note: If \( m_1(t) = m_2(t) \) (usual AM), spectrum at fundamental and together while at higher harmonics everything cancels!

Spectrum of \( x(t) \) \underline{not bandlimited} \( f_c \pm W \Rightarrow \text{won't work} \)

If you filter \( x(t) \) to make it bandlimited, essentially are modulating \( m_1(t) + m_2(t) \) and you can't separate them!

Patent denied!