a) If we multiply \( x(t) \cdot r(t) = AB \cos \omega_c t \cdot \cos(2\pi (f_c + \Delta f) t + \phi) \)

\[ = \frac{AB}{2} \left[ \cos(2\pi \Delta f t + \phi) + \cos(2\pi (2f_c + \Delta f) t + \phi) \right] \]

Lowpass filtering removes the high-frequency term. Need to perform a spectral analysis. Sample and compute FFT to find a spectral peak. The phase \( \phi \) won't affect the result.

b) The above scheme will not work for \( \Delta f \) negative.

Multiply by both \( \cos \omega_c t \) and \( \sin \omega_c t \).

\[ \cos \omega_c t \cdot \cos(2\pi (f_c + \Delta f) t + \phi) \rightarrow \text{LPF} \rightarrow \frac{1}{2} \cos(2\pi \Delta f t + \phi) \]

\[ \sin \omega_c t \cdot \cos(2\pi (f_c + \Delta f) t + \phi) \rightarrow \text{LPF} \rightarrow -\frac{1}{2} \sin(2\pi \Delta f t + \phi) \]

Now can form \( e^{j2\pi \Delta f t} \) and use spectral analysis (sample each component and calculate the FFT).

c) If you have multiple returns, the approach in (b) will work!